Pointers 5.5

Factorial

The factorial of a number (n!) is the product of all of the integers from that number down to 1.

6! = 6 x 5 x 4 x 3 x 2 x 1 = 720

You should have a built-in button or function on your calculator that computes factorials.

Ordering n objects

The number of ways that n objects can be put in order is n!

Multiplication Principle

If the first step of a choice can be made in m ways, and the second step of a choice can be made in n ways, then the entire choice can be made in m×n different ways.

Permutations

If we want to determine how many different ways we can order r objects selected from n objects, this is a permutation problem. Here is the formula.

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

You should have a button or menu item on your calculator that should compute this for you directly.

Suppose you want to compute 9P3. The numerator is 9!. To find the denominator, first subtract 9-3, then find the factorial.

$$_{9}P_{3} = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{362,880}{720} = 504$$

In the counting problems, the number we are selecting "out of" is n and goes before "P" in the notation.

The number we are selecting is r and goes after "P" in the notation. (Larger number is always first, smaller number is always last.)

Again, you should try to find the button or menu item on your calculator to do these directly. If you try it by hand, one important fact to remember is that 0! = 1. That one shows up every time n & r are the same number, and when we see 0! In the denominator we are tempted to try to divide by 0 but the denominator is actually 1.

Identifying Permutation Problems

There are 3 conditions we need for permutations.

1. Selecting r objects out of n distinct objects. (Look for "this many" out of "that many".)

2. No object can be selected more than once.

3. The order of selection matters. Being chosen first is different than being chosen second.

Example Suppose twenty runners are in a race. In how many different ways can they finish 1st,

2nd, and 3rd?

3 out of 20? Check!

Can a runner finish in 1st and 2nd? No – so that means that no object can be selected more than once.

Check!

Does the order matter? Finishing first is definitely different than finishing second. (Ask somebody who

got a silver medal in the Olympics if there is a difference between 1st and 2nd.) So, order matters. Check!

Now go to the formula or calculator.

$$ {}_{20}P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!} = 6840 $$

There are 6840 different ways that the runners can finish 1st, 2nd, and 3rd.

Combinations

Combinations are like permutations, except that we use combinations when the order of selection does not matter.

Here is the formula.

$$ {}_{n}C_{r} = \frac{n!}{r!(n-r)!} $$

Like permutations, you should also have a button or menu item on your calculator that should compute this for you directly.

Suppose you want to compute 9C3. The numerator is 9!.

To find the denominator, first subtract 9-3, which is 6. The denominator would then be 3! 6!.

Notice that there is an extra factor in the denominator.

$$ {}_9C_3 = \frac{9!}{3! \cdot (9-3)!} = \frac{9!}{3! \cdot 6!} = \frac{362,880}{4320} = 84 $$

If you try to do this directly on your calculator, type it out in the following order.

9!÷ 3!÷ 6!

You have to divide by both r! and (n-r)!.

Identifying Combination Problems

There are 3 conditions we need for combinations.

1. Selecting r objects out of n distinct objects. (Look for “this many” out of “that many”.)

2. No object can be selected more than once.

3. The order of selection does not matter. Being chosen first is NOT different than being chosen second.

Selecting a sample: Picked 1st or 2nd means you are in the sample.

Pizza toppings: Pepperoni & mushroom is the same pizza as mushroom & pepperoni.

Example Suppose a club has twenty members. In how many different ways can they choose 3 members to attend a conference?

3 out of 20? Check!

Since we need a total of 3 different club members to attend a conference, that means that no single member can be selected more than once. Check!

Does the order matter? No, the first person selected is going to the conference and the second person selected is also going to the conference – which is exactly the same result. So, order of selection does not matter. Check!

Now go to the formula or calculator.

$$ {}_{20}C_3 = \frac{20!}{3! \cdot (20-3)!} = \frac{20!}{3! \cdot 17!} = 1140 $$

There are 1140 different ways to select 3 club members to attend the conference.

Permutations or Combinations?

Basically, there are 3 conditions we need to check. The first 2 are the same for permutations and combinations. The 3rd is where the difference occurs.

1. Selecting r objects out of n distinct objects. (Look for "this many" out of "that many".)

2. No object can be selected more than once.

3. If the order of selection matters, use Permutations.

If the order of selection does not matter, use Combinations.

Using Combinations to Find Probabilities

We can use combinations to solve some probability problems we have already done using multiplication. You can do them whichever way you choose.

Example

Suppose you receive a shipment of 15 televisions, of which 5 are defective.

If you select 2 televisions at random, find the probability that at least 1 does not work.

Old Approach

We use the approach listed on the first page since we see the phrase "at least 1".

The complement of "TV does not work" is "TV does work".

$$\begin{aligned}
&P(\text{At least 1 does not work})\\
&=1-P(\text{0 do not work})\\
&=1-P(\text{Both work})\\
&=1-\frac{10}{15}\cdot\frac{9}{14}\\
&=1-\frac{3}{7}\\
&=\frac{4}{7}
\end{aligned}$$

New Approach – Using Combinations

We start the same way, but will use a different approach to find P(Both do work).

The numerator is the number of ways of selecting 2 of the 10 TVs that work: 10C2.

The denominator is the number of ways of selecting 2 of the 15 TVs: 15C2

$$
\begin{aligned}
&P(\text{At least 1 does not work}) \\
&= 1 - P(\text{0 do not work}) \\
&= 1 - P(\text{Both work}) \\
&= 1 - \frac{{}_{10}C_2}{{}_{15}C_2} \\
&= 1 - \frac{45}{105} \\
&= \frac{4}{7}
\end{aligned}
$$

You can choose either technique.

Example 2

You receive a shipment of 36 cell phones, of which 3 are broken. Your store's policy that the shipment will only be accepted if you take 4 phones from the shipment randomly and all 4 work. What is the probability that you will accept this shipment?

P(All 4 work) = Number of ways to select 4 good phones / Number of ways to select any 4 phones

Number of ways to select 4 good phones: 33C4

Number of ways to select any 4 phones: 36C4

P(All 4 work) = 33C4 / 36C4 = 40,920 / 58,905 = 248 / 357 = 0.6947

Old Way: 33/36 x 32/35 x 31/34 x 30/33