HW 5.4 Guide

1: This question makes sure that you know the notation that is used for conditional probabilities.

Help with Parts a & b

a) What is the probability that a randomly selected individual is at least 55 years of age, given the individual is neither more nor less likely to buy a product emphasized as "Made in our country"?

This is an example of a conditional probability.
You are given a condition that reduces the entire table to particular rows or columns.
When it says "given that the individual is neither more nor less likely …", that tells us to work only with the row labeled "Neither more nor less likely". That means that the denominator will be 802.

Purchase likelihood	18-34	35-44	45-54	55 +	Total
More likely	225	333	338	405	1301
Less likely	29	6	22	18	75
Neither more nor less likely	288	212	168	134	802
Total	542	551	528	557	2178

We were asked to find the probability that an individual is at least 55 given that condition.
How many of those 802 people were at least 55 years old?

Purchase likelihood	18-34	35-44	45-54	55 +	Total
More likely	225	333	338	405	1301
Less likely	29	6	22	18	75
Neither more nor less likely	288	212	168	134	802
Total	542	551	528	557	2178

134 out of 802, so the probability is 134/802 or approximately 0.167.

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b) What is the probability that a randomly selected individual is neither more nor less likely to buy a product emphasized as "Made in our country," given the individual is at least 55 years of age?

In this problem the given condition is that the individual is at least 55 years of age, so we work only with the column "55+".

Purchase likelihood	18-34	35-44	45-54	55 +	Total
More likely	225	333	338	405	1301
Less likely	29	6	22	18	75
Neither more nor less likely	288	212	168	134	802
Total	542	551	528	557	2178

Of the 557 people in that column, how many are neither more nor less likely to buy …?
There are 134 people in that row, so the probability is 134/557 or approximately 0.241.

2: Compute a conditional probability using the formula.

3: Use the conditional probability formula in reverse to find P(E and F).

4: Determine whether the two events are independent.

5-6: Conditional Rule

7: Use the multiplication rule and “at least 1” strategies from the last section The only difference is that when we deal with dependent events the probability changes from trial 1 to trial 2.

Suppose you just received a shipment of ten televisions. Four of the televisions are defective. If two televisions are randomly selected, compute the probability that both televisions work. What is the probability at least one of the two televisions does not work?

Part A: Both Work
4 of the TV’s are defective (D) and the other 6 work (W), so the shipment looks like this: DDDDWWWWWW

P(Both Work)
=P(W 1st AND W 2nd)
=P(W 1st) x P(W 2nd)
=6/10 x 5/9
=30/90
=1/3

The reason the first probability is 6/10 is because there are 6 working TV’s out of the shipment of 10. The reason the second probability is 5/9 is that once the first working TV is selected there are only 5 working TV’s left of the 9 that remain: DDDDWWWWW

Part B: At Least 1 Does Not Work
In this part we have to use the ideas of “at least 1” from 5.3.
P(At least 1 is defective)
=1 – P(0 are D)
=1 – P(2 Work)
=1 – 0.333 [from Part A]
=0.667

8: Sampling without replacement means that the first card is selected and put aside. (Dependent) Sampling with replacement means that the first card is reinserted into the deck before the second card is shuffled. (Independent)

9:

P(You like both) = P(Like 1^{st}) x P(Like 2^{nd})

P(You don't like both) = P(Don't like 1^{st}) x P(Don't like 2^{nd})

There are 2 ways to like exactly 1. Either you like the first but not the second, or you don't like the first and like the second. Figure each probability and add because we are looking for the probability of (like 1, don't like 2) **OR** (don't like 1, like 2).

In part d, repeat these problems using sampling with replacement.

Suppose a compact disk (CD) you just purchased has 10 tracks. After listening to the CD you decide that you like 4 of the songs. With the random feature on your CD player, each of the 10 songs is played once in random order. Find the probability that among the first two songs played

(a) You like both of them. Would this be unusual?

I will use "L" for like and "D" for do not like. So the CD looks like this: LLLL DDDDDD

P(Like Both) = P(L 1st) x P(L 2nd) = 4/10 x 3/9 = 12/90 = 0.133

The 3/9 comes from the fact that there is 1 less song that you like (4-1=3) and 1 less song (10-1=9).

Keep in mind that an event is unusual only if its probability is less than 0.05.

(b) You like neither of them.

P(Like neither) = P(Dislike both) = P(D 1st) x P(D 2nd) = 6/10 x 5/9 = 0.333

This time you start with 6 out of 10 that you do not like, and then chop that down to 5 out of 9 for the second song.

(c) You like exactly one of them.

There are 2 ways for this to happen – either you like the first and dislike the second (LD) or you dislike the first and like the second (DL). You have to figure out each one separately, then add the answers.

Like first and dislike second:

P(LD) = 4 L's / 10 songs x 6 D's / 9 songs = 24/90 = 0.267

Notice that the denominator got smaller, but the numerator switched from L's to D's.

Dislike first and like second:

P(DL) = 6/10 x 4/9 = 24/90 = 0.267

Add: 0.267 + 0.267 = 0.534

(d) Redo (a)-(c) if a song can be replayed before all 10 songs are played.

For the next three problems, since a song can replayed the probabilities do not change from the first song to the second song, we are always using LLLL DDDDDD

For instance part (a) will switch from 4/10 x 3/9 to 4/10 x 4/10.

10-12: Similar to previous exercises.

11: Example

Determine the probability that at least 2 people in a room of 10 people share the same birthday, ignoring leap years and assuming each birthday is equally likely, by answering the following questions:

(a) Compute the probability that 10 people have different birthdays.

Starting with 365 different days of the year, the first person could have a birthday on any of them.
The second person could not match the first person, so only 364 of the 365 days will work.
The third person cannot match either of the first two, so we are down to 363 out of 365.
Keep going until you get to 10 people, then multiple until you get to 10 people.
365/365 x 364/365 x 363/365 x 362/365 x …

(b) The complement of "10 people have different birthdays" is "at least 2 share a birthday". Use this information to compute the probability that at least 2 people out of 10 share the same birthday.

This is just an application of the complement rule using the answer from part (a).

13: Make use of our work with contingency tables in an earlier section.
A and B are independent is $P(A) = P(A|B)$