

Pointers Section 5.3

Events “A” and “B” are independent events if P(A) is not affected by whether or not event B occurs or not. In other words, event A is unaffected by event B.

Multiplication Rule for Independent Events

If A and B are independent events, then $P(A \text{ and } B) = P(A) \times P(B)$

“AND” typically leads to MULTIPLICATION

If there are 3 or more events, and you want to find the probability that all of the events occur, then you multiply the probabilities for each of the events occurring.

Suppose the probability that a child likes broccoli is 0.05, and we want to find the probability that 2 randomly selected children like broccoli. The events are independent, because one child’s feelings about broccoli do not affect another child’s feelings about broccoli.

$$\begin{aligned} &P(\text{1st likes broccoli AND 2nd likes broccoli}) \\ &= P(\text{1st likes broccoli}) \times P(\text{2nd likes broccoli}) \\ &= (0.05)(0.05) \\ &= 0.0025 \end{aligned}$$

If you randomly select 5 children and wanted to find the probability that all 5 like broccoli, that would be equal to $(0.05) (0.05) (0.05) (0.05) (0.05)$ or $0.05^5 = 0.000003125$.

If you select 2 children and want to find the probability that they both do not like broccoli, first note that the probability that a child does not like broccoli is $1 - 0.05 = 0.95$.

$$\begin{aligned} &P(\text{1st does not like broccoli AND 2nd does not like broccoli}) \\ &= P(\text{1st does not like broccoli}) \times P(\text{2nd does not like broccoli}) \\ &= (0.95)(0.95) \\ &= 0.9025 \end{aligned}$$

“Interpretation” – In these problems, look at the number of trials that MML gives you (100, 1000, ...). For 100 trials, round the probability to the nearest 100th, and you will see the expected number of successes in 100 trials. For 1000 trials, round to the nearest 1000th, ...

Probability of “at least 1 success”.

These problems are based on the multiplication rule.

If there are 5 trials of an experiment and we are interested in the probability of at least one “success”, that means we are looking for 1, 2, 3, 4, or 5 “successes”.

That includes all of the possibilities of 0 “successes”.

We can use the complement rule by first finding the probability of 0 successes, and then subtracting that answer from 1.

Example The probability that a COS student is female is 0.6, and we randomly select 4 COS students. Suppose that we want to find the probability that at least 1 of the students is female.

By definition, that probability will be equal to $1 - P(0 \text{ females})$.

In this problem, “0 females” means “4 males”.

We need to know the probability that a student is a male. Since $P(\text{Female}) = 0.6$, $P(\text{Male}) = 1 - 0.6 = 0.4$.

Here are the steps to solve this problem:

$$\begin{aligned} &P(\text{At least 1 female}) \\ &= 1 - P(0 \text{ females}) \\ &= 1 - P(4 \text{ males}) \\ &= 1 - (0.4)(0.4)(0.4)(0.4) \\ &= 1 - 0.0256 \\ &= 0.9744 \end{aligned}$$

Example 20% of all students have an iPhone. If 5 students are selected at random, find the probability that at least 1 student has an iPhone.

$$\begin{aligned} &P(\text{At least 1 iPhone}) \\ &= 1 - P(0 \text{ iPhones}) \\ &= 1 - P(\text{all 5 do not have an iPhone}) \\ &= 1 - (0.8)(0.8)(0.8)(0.8)(0.8) \\ &= 1 - 0.32768 \\ &= 0.67232 \end{aligned}$$

***AT LEAST 1 tells us to find $1 - P(0)$
0 iPhones means all 5 do not have an iPhone
If 20% have an iPhone, 80% do not. So we will multiply
0.8 as a factor 5 times (for 5 students).***