Pointers Section 5.4

Conditional Probability

A conditional probability means that some information is known that restricts the original sample space in some way. For example, the probability of you getting a speeding ticket this semester changes if we know that you drive a lot of miles or that you typically drive very fast. If there is a 5% chance that a student gets a speeding ticket, that probability will decrease if you live 3 blocks from the college and it will increase if you have a lead foot.

Notation: The conditional probability of "A" occurring given that "B" has occurred is written P(A|B).

Formula: $P(A \mid B) = \dfrac{P(A \text{ and } B)}{P(B)}$

We divide the probability of both occurring by the probability of the given event.

In some problems, you will be given 2 of the following 3 probabilities:

P(E), P(E and F), and P(F|E)

Using the formula $P(F \mid E) = \dfrac{P(E \text{ and } F)}{P(E)}$, plug in the two probabilities you know and solve for the missing one.

Example: P(E) = 0.2, P(E and F) = 0.1. Find P(F|E).

$$P(F \mid E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{0.1}{0.2} = 0.5$$

Example: If a card is drawn from a deck of 52 cards, find the probability that it is a heart.

Divide the number of "hearts" by the number of "cards".

Since there are 13 hearts in a deck, the probability is 13/52 or ¼.

However, if the problem changes so that we are told that we are told that the card is red, the answer will change.

P(Heart | Red) = P(Heart AND Red) / P(Red)

There are 13 cards that are red hearts, and 26 cards that are red, so the probability is 13/26 or ½.

Example: 60% of COS students are female, and 15% of COS students are female and own an iPhone.

If a female student is selected at random, find the probability that she owns an iPhone.

Solution

Let "F" represent the event that a student is Female, and "I" represent the event that a student owns an iPhone.

This is a conditional probability problem because it starts out with "If a female student is selected at random" rather than "If a COS student is selected at random".

Start by labeling the probabilities given in the problem.

- 60% of COS students are female: P(F) = 0.60
- 15% of COS students are female and own an iPhone: P(F and I) = 0.15

Now label the probability we are seeking in symbols.

- "If a female student is selected at random": F is the given event
- "find the probability that she owns an iPhone": I is the probability we are looking for
- We want P(I | F). The given event goes after the vertical line segment, the event we want the probability of goes in front of it.

Set up the probability formula: $P(I \mid F) = \dfrac{P(Both)}{P(Given)} = \dfrac{P(I \text{ and } F)}{P(F)}$

Now plug in: $P(I \mid F) = \dfrac{P(Both)}{P(Given)} = \dfrac{P(I \text{ and } F)}{P(F)} = \dfrac{0.15}{0.60} = 0.25$

We use multiplication whenever we are looking for the probability of A and B, or “ALL” or “NONE”.

Multiply the probability of the first event occurring by the probability of the second event occurring, …

For the second event we often have to adjust the probabilities because this is a conditional probability situation, depending on what happened in the first trial.

“At Least One”

Suppose we surveyed 8 students and wanted to find the probability that at least 1 of the students had an iPhone. To do this directly, we would have to find the probability that 1 student does, add that to the probability that 2 students do, + … + the probability that 8 students do. That is a difficult task, and we can solve this in a more efficient way by using the complement rule.

Notice that at least 1 includes all of these possibilities: 1, 2, 3, 4, 5, 6, 7, and 8. Those are all of the possibilities except for 0 students having an iPhone.

Here are the steps, if we let A be the event that a student has an iPhone.

$$P(\text{At least 1 } A) = 1 - P(0\ A) = 1 - P\left(A^C\right)$$

Example 1

Suppose you receive a shipment of 15 televisions, of which 5 are defective. If you select 2 televisions at random, find the probability that both work.

What is the probability that the first television works?

- If 5 TV’s are defective, that means that the other 10 (15 – 5) work. So, the probability that the first TV works is 10/15.

What is the probability that the second TV works?

- There are only 14 TV’s left to choose from, and since we already picked a good one that leaves 9 TV’s that work. So the probability is 9/14.

$$P(\text{Both Work}) = \frac{10}{15} \cdot \frac{9}{14} = \frac{3}{7}$$

If you select 2 televisions at random, find the probability that at least 1 does not work.

We use the approach listed on the first page since we see the phrase "at least 1".

The complement of "TV does not work" is "TV does work".

$$
\begin{aligned}
P(\text{At least 1 does not work}) &= 1 - P(\text{0 do not work}) \\
&= 1 - P(\text{Both do work}) \\
&= 1 - \frac{10}{15} \cdot \frac{9}{14} \\
&= 1 - \frac{3}{7} \\
&= \frac{4}{7}
\end{aligned}
$$

In both of these problems, divide your final answer to get a decimal result for MML.

Problem 2

Sampling without replacement: After the first card is selected, it is thrown away leaving only 51 cards in the deck.

Sampling with replacement: After the first card is selected, it is put back into the deck meaning that we still have 52 cards when picking the second card.

Suppose 2 cards are drawn from a deck. What is the probability that both cards are Queens if the sampling is done without replacement?

P(First card is a Queen) = 4/52.

P(Second card is a Queen) = 3/51 because there is 1 less Queen and 1 less card in the deck.

$$P(\text{Q first AND Q second}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$$

Suppose 2 cards are drawn from a deck. What is the probability that both cards are Queens if the sampling is done with replacement?

P(First card is a Queen) = 4/52.

P(Second card is a Queen) = 4/52 because the deck is unchanged.

$$P(\text{Q first AND Q second}) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$$

Problem 3

This problem involves a CD with 15 songs, of which you only like 4. Your CD player is set to random. The question regards the first 2 songs played.

L: Like (4 of 15), D: Don’t Like (11 of 15)

a) P(You like both)

P(Like first) = 4/15

P(Like second) 3/14

P(Like both) = 4/15 x 3/14 = 2/35

Convert that to a decimal, and if it less than .05 we consider that an unusual event.

b) P(You don’t like both)

P(Don’t like first) = 11/15

P(Don’t like second) 10/14

P(Don’t like both) = 11/15 x 10/14 = 11/21

c) P(You like exactly 1)

In part a) we found the probability you like 2. In part b) we found the probability you like 0. The only other option is liking exactly 1. Just subtract the answers from parts a) and b) from 1.

Problem 4

You have a bag of tulip bulbs: 30 are red, 10 are yellow, 20 are purple. You select 2 bulbs from the bag.

a) Find the probability that both are red.

P(Both Red) = P(1st Red) x P(2nd Red) = 30/60 x 29/59 = 29/118

b) Find the probability that the first is red and the second is yellow.

Since the first bulb is red, that still leaves 10 yellow bulbs in the bag that now contains only 59 bulbs.

P(1st Red) x P(2nd Yellow) = 30/60 x 10/59 = 5/59

c) Find the probability that the first is yellow and the second is red.

Same idea, opposite order.

P(1st Y) x P(2nd R) = 10/60 x 30/59 = 5/59

d) Find the probability that exactly one is red and one is yellow.

There are 2 ways this can happen: Red/Yellow or Yellow/Red.

Add the probabilities from parts b & c.